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Modal Parameter Identification Using Simulated Evolution

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A global optimization algorithm based on the Darwinian evolutionary theory is implemented for the identification of modal parameters of linear vibrating structures. The capability of the evolutionary search method in locating the global minimum among numerous local minima is demonstrated. The modal parameter identification procedure is based on the concept of modal sweep, according to which modes of a structure are identified individually and iteratively. A fast recursive algorithm is also developed for forward response calculation of a linear oscillator to lessen the computational burden of using simulated evolution in the iterative identification process. The proposed modal parameter identification approach does not require any initial estimate of parameter values. Simulation results show that the approach is effective and reliable in identifying dominant modes under noisy conditions.

Introduction

TO effectively control a system, it is necessary to have a mathematical model that can adequately describe the physical and dynamic characteristics of the system. Analytical models are usually built under assumptions of some ideal conditions, which are different from the realistic conditions, so the model may fail to adequately represent the true system. Therefore, the use of test data usually plays an important part in obtaining an appropriate model. Through experimentally observed data of the system, parameters of the mathematical model can then be estimated. System identification deals with such a problem of designing and optimizing mathematical models with measured data.

Once a class of models has been chosen, the identification problem is reduced to a parameter estimation problem, in which the optimization search is usually performed over the response surface defined by a cost function. In most inverse engineering problems, response surfaces are highly nonlinear and may consist of numerous local minima. Classic optimization techniques such as the Powell method or the gradient method[1] can locate the globally optimal solution only if the response surface being investigated is unimodal or if the initial guess chosen is near the global minimum. Otherwise, the optimization search may be trapped in an undesired local optimum, which may result in an inadequate model. Therefore, an optimization technique that can locate the global minimum is highly desirable for such problems. The process of natural evolution provides us with such a search mechanism that will be discussed later.

There are many advantages of using models based on the dominant modes in a structural identification problem. The modal models deal directly with the parameters that control the structural output, and they are identifiable in most cases.[2] Many techniques have been developed for modal parameter identification in the past, including both the frequency-domain methods and the time-domain methods. The frequency-domain methods[3,4] require evaluation of the frequency response functions from the measured data and then identify modal parameters from the frequency response functions. Ibrahim and Mikulcik[5] introduced a time-domain method, which identifies the modal parameters by solving an augmented eigenvalue problem. However, it applies only to problems involving response data of free vibration. Juang and Pappa[6] developed the eigensystem realization algorithm based on the singular value decomposition of the generalized Hankel matrix, which is composed of discrete, time-domain measurements. However, difficulties with the eigensystem realization algorithm may arise when high noise levels are present in the measurements.

The problem of modal parameter identification can also be considered an optimization problem. The accuracy of identification results strongly depends on the optimization techniques employed. As mentioned earlier, it is desirable that the optimization technique can locate the global minimum in spite of the presence of multiple local minima. The process of natural evolution provides us with such a technique. In the present study, we investigate the applicability of the method of simulated evolution to modal parameter identification of linear vibrating structures. The robustness of the proposed approach in identifying modal parameters of a linear system from high-noise measurements is examined.

Method of Simulated Evolution

Natural evolution is a learning process in which the behavior of individual organisms is an inductive inference concerning some aspects of their environment. Validation is demonstrated through their survival. Over successive generations, organisms become better and better predictors of their surroundings. The most generally accepted theory of evolution was presented by Darwin and Wallace jointly in 1858 (Ref. 7). According to the theory, the natural evolution process consists of four major steps: 1) reproduction, 2) mutation, 3) competition, and 4) selection. The reproduction process is the replication of the parents themselves, which is a property of almost all living organisms. In the realistic world, error in reproduction is inevitable. Such replicative error is a fundamental of the mutation process and is necessary to the evolutionary process. Competition is a consequence of expanding populations of organisms in a finite space. Selection is the result of competitive replication as organisms fill the available space.

Bremermann[8] recognized that biological evolution is an optimization process. The vector of design variables is considered to be an organism. The components of this variable vector were claimed to be analogous to an organism's genes. Fogel and Atmar[9] studied evolutionary mechanisms based on simulated sexual recombination. Their results indicated that modifying each component of the evolving solution by a Gaussian random variable results in an efficient search for global optima. The method of simulated evolution is similar to genetic algorithms,[10] which are also based on Darwin's theory of survival of the fittest. The genetic algorithms, however, involve the encoding and crossover operations on the design variables, which are avoided by the method of simulated evolution without losing the effectiveness and efficiency in locating global optima.[9]

Implementation of the Evolutionary Search for Function Optimization

The method of simulated evolution is implemented here for function optimization problems to demonstrate its capability in locating global optima of complex functions. In the process, the variable vector takes the role of an organism so that each point in the parameter space is considered to be an organism. The function value of this point is analogous to the fitness of the organism to the current environment. At the beginning of each generation, each organism

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(variable vector) reproduces itself to give an offspring to which a (random) replicative error is added to account for mutation. The organisms of the two generations compete with one another according to a given competition rule. It is remarkable that competition for survival should be "probabilistic" in nature to reduce the chances of settling for a false optimum. Each organism competes with randomly selected organisms in the whole population to obtain a fitness score. The organisms with the highest scores survive as parents for the next generation and the rest are eliminated. The same process repeats until the whole population is well evolved. The evolutionary search is implemented as follows.[11]

1) An initial population of trials is chosen at random by setting $p_i = P_i \sim U(a, b)^n$, $\forall i = 1, \ldots, k$, where P_i is a random vector, p_i is the outcome of the random vector and represents the ith organism of the population. Each component of the vector p_i represents a variable of the function optimized. $U(a, b)^n$ denotes a uniform distribution ranging over $[a, b]$ in n dimensions (n = number of variables), and k is the initial population size, whose value is generally much larger than one.

2) The function value $F(p_i)$ of each p_i, $i = 1, \ldots, k$, is calculated.

3) Each p_i, $i = 1, \ldots, k$, produces an offspring vector p_{i+k} with random replicative error. Denoting $p_{i,j}$ as the jth element of the ith organism, we define $p_{i+k,j} = p_{i,j} + N(0, \sigma^2)$, $\forall j = 1, \ldots, n$, where $N(\mu, \sigma^2)$ represents a Gaussian random variable with mean μ and variance σ^2.

4) The function values of the newly produced k organisms, $F(p_i)$, $i = k + 1, \ldots, 2k$, are calculated. Each of the $2k$ organisms undergoes competitions (function value comparisons) with m randomly chosen organisms.

5) The first k organisms having the most "wins" survive as parents for the next generation, and the remaining k organisms with fewer wins are eliminated.

6) The process repeats by returning to step 3. A halt to the sequence is declared when either a predetermined quality of solution has been achieved or a set number of iterations has been exhausted.

Minor variations of this basic procedure are possible, but these steps are general characteristics of all evolutionary algorithms. Evolutionary programming has been applied to many engineering problems, such as the traveling salesman problem[12] and the neural network training problem.[13] The success of evolutionary programming suggests that simulated evolution can be a useful technique for estimating globally optimal parameters of a model.

Numerical Example

To examine the algorithm of evolutionary search, we consider the following 10-dimensional function[14]:

$$F(x_1, \ldots, x_{10}) = 0.1 \times \left\{ \sum_{i=1}^{9} \left[x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i)\cos(4\pi x_{i+1}) + 0.3\right] + x_{10}^2 + 2x_1^2 - 0.3\cos(3\pi x_{10})\cos(4\pi x_1) + 0.3 \right\}$$

which is actually the summation of 10 two-dimensional functions. The function has a single global minimum at $x_1 = \ldots = x_{10} = 0$. The two-dimensional version of the function is illustrated in Fig. 1 for $x_i, x_j \in [-1, 1]$, which shows numerous local minima around the global minimum. In this example, an initial population of 50 variable vectors, i.e., $k = 50$, are generated at random, with each component varying uniformly over the range $[-50, 50]$. One offspring vector is produced from each parent vector by adding a Gaussian random variable with a zero mean and a standard deviation for mutation. During competitions, each organism obtains a competition score in accordance with the competition rule given in step 4, in which the number of competitors chosen for each organism equals 10, i.e., $m = 10$.

Evolutionary search is applied to the function using 1) a constant standard deviation of 5 (for mutation) and 2) an adjustable standard deviation of initial value 15. The motivation for employing adjustable deviations is to improve the convergence rate of evolutionary search. The rule for adjustment of the standard deviation is proposed as follows: if the objective function value of the "best" organism remains the same for five consecutive generations, then the standard deviation is decreased by half.

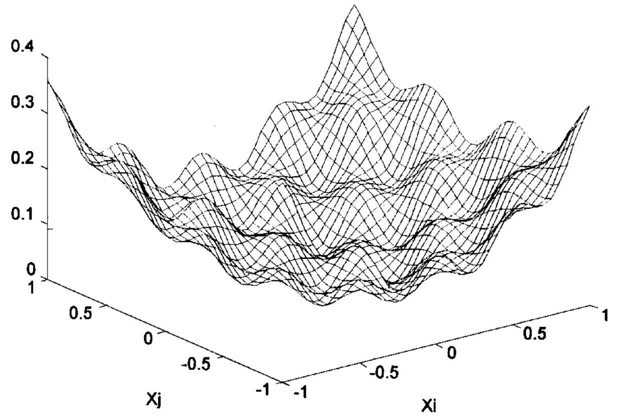
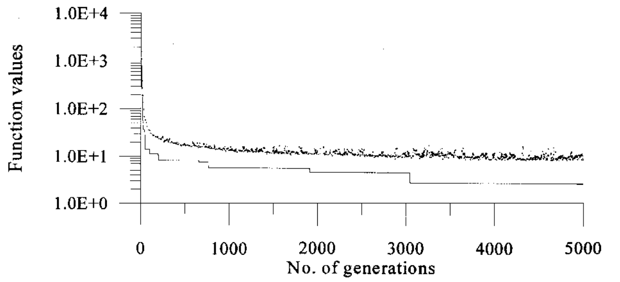
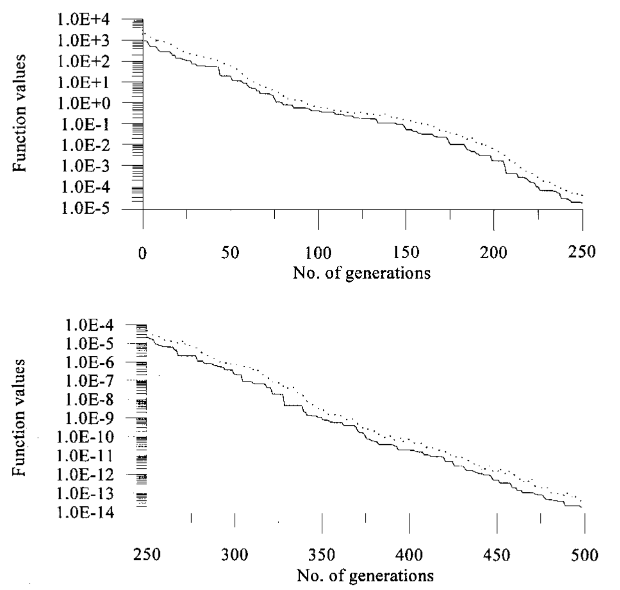


Fig. 1 Ten-dimensional function with multiple optima (two-dimensional version); $x_i, x_j \in [-1, 1]$.



a) Standard deviation = 5%



b) Adjustable standard deviation

Fig. 2 Histories of selected organisms over successive generations for the 10-dimensional function: ——, best organism and ....., worst organism.

Evolution is halted after 5000 and 500 generations, respectively, for cases 1 and 2. Figure 2 shows the histories of the function values for these two cases over successive generations. Figure 2a shows that the best organism of the population gradually moves toward a location with relatively low function value, but, after a certain number of generations, the population fails to produce a better organism for many consecutive generations. Also, the function value of the worst organism remains large, which indicates that the population still scatters over the domain. On the other hand, Fig. 2b corresponds to the case with adjustable deviations and shows that the function values decrease rapidly over successive generations and the two curves that represent the function values of the best and the worst organisms get closer and closer as a generation evolves. This implies that the whole population eventually concentrates around the global minimum. Comparing the results obtained in these two cases, we conclude that an evolutionary search using an adjustable standard deviation for mutation is generally more efficient and more accurate. It should be remarked, however, that no claim is made about the optimality of the adjustment rule proposed. In general, the standard deviation for mutation should decrease slowly as the generations evolve or as the function values decrease.

Modal Parameter Identification of Linear Structures

The choice of model structure is often crucial in identifying an appropriate representation for a system. Once a set of models has been chosen, a search is initiated for the best model in that set according to some criterion. Under certain circumstances, however, a model thus obtained can be deficient because 1) the observation data collected for identification purposes are not informative enough, 2) the chosen range of models may not be appropriate in the sense that it does not provide an adequate description of the system, 3) the optimization method for estimating the parameters associated with the model is not powerful enough to locate an optimal set of values in light of a specified criterion, or 4) the convergence criterion itself is inappropriate. To avoid or overcome these difficulties, a modal model is employed for identification of linear structures. A central point of the present study is to examine the applicability of the method of simulated evolution to modal parameter identification.

Fast Recursive Algorithm for Response Calculation

Consider an N-degree-of-freedom linear vibrating structure. The equation of motion of the system can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{r}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ represent the mass, damping, and stiffness matrices, respectively; $\boldsymbol{r}(t)$ is the $N \times 1$ excitation vector; and $\boldsymbol{x}$ is the $N \times 1$ displacement vector. If the system has proportional damping or, more generally, has classic normal modes[15] and is excited by a base excitation $\ddot{y}(t)$, then by introducing the coordinate transformation

$$\boldsymbol{x} = \Phi \boldsymbol{q} \tag{2}$$

where Φ is the system eigenvector matrix and $\boldsymbol{q}$ is the generalized displacement vector, Eq. (1) can be decoupled into

$$\ddot{q}_i + 2\omega_i \varsigma_i \dot{q}_i + \omega_i^2 q_i = -\ddot{y}(t) \sum_j^N \phi_{ji} m_j \qquad i = 1 \sim N \tag{3}$$

in which ω_i and ς_i are the natural frequency and modal damping of the ith mode, respectively; ϕ_{ji} is the jth component of the ith mode shape; and m_j is the lumped mass of the jth degree of freedom. Multiplying both sides of Eq. (3) by ϕ_{ki}, we can get

$$\ddot{x}_{ki} + 2\omega_i \varsigma_i \dot{x}_{ki} + \omega_i^2 x_{ki} = -P_{ki}\ddot{y}(t) \tag{4}$$

where $x_{ki} = \phi_{ki} q_i$ represents the contribution of the ith mode to the response at the kth degree of freedom and P_{ki} is called the effective participation factor.[16]

The response of a linear structure to some excitation can be computed by solving Eq. (4) for each of the dominant modes and then superimposing the modal responses. In view of the large amount of computations involved in the optimization process using an evolutionary search, it is desirable to have a highly efficient solver for Eq. (4) if modal identification is to be performed in the time domain based on the concept of modal sweep.[16] Motivated by a fast algorithm proposed by Beck and Dowling[17] for forward response calculation of a linear oscillator, which assumed linear interpolation between two consecutive time instants of digitized input data, we develop a more accurate version by assuming quadratic interpolation among three consecutive time instants of input data. The quadratic version retains the same computational efficiency but is more accurate than the linear version. According to the quadratic algorithm, the acceleration response a_i at the ith time step can be calculated by the following recursive algorithm:

$$\begin{aligned} a_{i+1} &= b_1 a_i + b_2 a_{i-1} + e_1(r_{i-1} - 2r_i + r_{i+1}) \qquad i \geq 1 \\ a_1 &= b_3 x_0 + b_4 v_0 + e_2 r_0 + e_3 r_1 \end{aligned} \tag{5}$$

where

$$b_1 = 2e^{-h\Omega_0}\cos\Omega_d \qquad b_2 = -e^{-2h\Omega_0}$$

$$b_3 = \omega_0^2 e^{-h\Omega_0}\left(\frac{h\Omega_0}{\Omega_d}\sin\Omega_d - \cos\Omega_d\right)$$

$$b_4 = -\omega_0 e^{-h\Omega_0}\left[\frac{(1-2h^2)\Omega_0}{\Omega_d}\sin\Omega_d + 2h\cos\Omega_d\right]$$

$$e_1 = \frac{1}{\Omega_0^2} - \frac{2}{\Omega_0^2}\cos\Omega_d e^{-h\Omega_0} + \frac{1}{\Omega_0^2}e^{-2h\Omega_0}$$

$$e_2 = e^{-h\Omega_0}\left(\cos\Omega_d - \frac{1+h\Omega_0}{\Omega_d}\sin\Omega_d\right) \qquad e_3 = \frac{1}{\Omega_d}e^{-h\Omega_0}\sin\Omega_d$$

$$\Omega_0 = \omega_0\Delta t \qquad \Omega_d = \omega_0\sqrt{1-h^2}\Delta t \qquad (\Delta t \text{ is the sampling time})$$

In the preceding equation, ω_0 and h denote the natural frequency and the damping ratio of a linear oscillator, respectively, and r_i is the excitation at the ith instant. Note that the initial acceleration a_0 is obtainable from the equation of motion given initial displacement x_0, initial velocity v_0, and r_0.

From Eq. (2), it is clear that the acceleration response of the kth degree of freedom equals the sum of all a_{ki} for $i = 1, \ldots, N$. The time history of each a_{ki} can be calculated by Eq. (5). It can be shown that the quadratic algorithm is much more efficient (only three multiplication and a few summation operations are needed for response at each time step) and accurate than conventional integration techniques, such as the fourth-order Runge–Kutta method.[18]

Concept of Modal Sweep

For the modal parameter identification problem of interest, the cost function to be minimized can be chosen as the squared output error

$$J = \sum [a_k(i,\theta) - \tilde{a}_k(i)]^2 \tag{6}$$

where $a_k(i,\theta)$ and $\tilde{a}_k(i)$ represent the modeled and measured acceleration response, respectively, and θ denotes the parameter vector. To identify the modal parameters, we introduce the concept of modal sweep.[16] Minimization of J is achieved by a succession of modal sweeps, with each sweep involving M single-mode minimizations to get new estimates for the parameters of each of the M modes in the model. Thus, J is first minimized with respect to the modal parameters of only one mode. After reaching a tolerable fitness of measure, a new mode is added to the model so that the minimization is then performed with respect to the modal parameters of two modes. The process repeats until the fractional decrease in J is less than a specified value. Further addition of vibration modes to the model is justified by some criterion to be described later.

Convergence Criterion

It has been demonstrated that the evolutionary search technique is capable of locating global minima in spite of the presence of numerous local minima. However, one might have noticed that, in the previous example of function optimization, no convergence criterion was used. Conventionally, an optimization search is considered to be converged if the absolute or the relative change in the value of the cost function is smaller than a specified tolerance. Unfortunately, during an evolutionary search, the objective function values of the best organism may remain the same for several generations, and hence the conventional convergence criteria may lead to a false solution that is not even a local minimum.

According to the concept of "adaptive landscape,"[19] the organisms will move toward the extremum of the landscape (a location corresponds to best fit to the environment). A new criterion is proposed based on this idea. The evolutionary search is considered to be converged if the difference in "shape" (or the Euclidean distance) between the best and worst organisms is smaller than a specific value, i.e.,

$$\left| x_k^B - x_k^W \right| \leq \varepsilon_a \qquad \text{for} \qquad k = 1, \ldots, n \tag{7}$$

where x_k^B and x_k^W represent the kth component of the best and the worst parameter vectors, respectively. For the case of modal parameter identification using modal sweep, an extra mode is added to the modal model only if condition (7) is satisfied for the current model.

Techniques for Assessing Model Order

Determination of model order is an important aspect in system identification. An overparametrized model will lead to unnecessarily complicated computations during parameter estimation and unreliable results may be obtained. On the other hand, an underparametrized model may be quite inaccurate. To determine an appropriate model order, a criterion is usually used to penalize the decrease of the cost function with increasing model size. The model structure giving the smallest value of this criterion will be accepted. A general form of this criterion is[20]

$$W_N = N \log J_N^{(p)} + \gamma(N, p) \tag{8}$$

where $J_N^{(p)}$ represents the value of the loss function defined by the p-parameter model using N data points of observation and $\gamma(N, p)$ is a function of the number of samples N and the number of parameters p in the model. The term $\gamma(N, p)$ penalizes overparametrized models in view of the parsimony principle, which states that of some competing models that all explain the data well, the model with the smallest number of independent parameters should be chosen. The choices $\gamma(N, p) = 2p$ and $\gamma(N, p) = p \log N$ will give, respectively, the widely used Akaike's information criterion (AIC)[21] and the minimum defining length (MDL) principle.[22] The AIC is a statistical technique incorporating both the maximum-likelihood principle and the parsimony principle; the MDL principle asserts that the optimal model size is that yielding the shortest description of the observed data in an information sense.

A model selection criterion is said to be consistent if the probability of selecting a wrong (overparametrized or underparametrized) model tends to zero as the number of data points tends to infinity. It can be shown that the MDL criterion provides a consistent estimate of model order and hence is more reliable than the AIC if the number of data points available is large.[20]

Numerical Simulation

Consider a simple 10-degree-of-freedom chain mass-spring-damper system, which has been studied by Beck and Jennings[16] with a six-mode model. Consider that the system is excited by a simulated zero-mean bandpass white noise,[23] which can effectively excite all vibration modes of interest. The frequencies of the excitation signal range from 0 to 80 rad/s, and the standard deviation is equal to 2.23 ms^{-2}. The duration and sampling time of the signal are 10.2 and 0.04 s, respectively. Only simulated data of the "roof" response (at the 10th degree of freedom) and base excitation record are used for modal parameter identification of the 10-degree-of-freedom system.

In practical problems, because of uncertainties of experimental conditions, environmental disturbances, and quality of measuring equipment, measurement noise is unavoidable from the desired signals. In the simulation, we consider the presence of measurement noise of a relatively high level. The noise level is measured by the signal-to-noise ratio (SNR), which is defined as the standard deviation of the signal divided by that of the noise. To examine the robustness of the proposed identification approach under noisy conditions, two cases with SNR equal to 5 and 1, respectively, are investigated.

The modal parameter identification results using modal sweep combined with simulated evolution are given in Table 1. For both cases, the first four vibration modes are successfully identified through an evolutionary search that terminated after 5000 generations. The results for the case with lower noise (SNR = 5) are generally better than those for SNR = 1, as expected. The errors of the identified natural frequencies are lower than the errors in modal dampings and effective participation factors (which are equivalent to the components of mode shapes at the locations of measurement). The high percentage of errors in the modal dampings and in the effective participation factors for higher modes can be attributed to the presence of high noise and to the low response sensitivity of the two parameters. Note, however, that the results identified by simulated evolution could have been better if a stricter convergence criterion was employed and a higher number of generations was allowed before termination.

The values of the minimized cost function, AIC, and MDL of the models having different numbers of modes are shown in Table 2, which indicates that a four-mode model is appropriate for both cases. It is noted, however, that for the case with higher noise (SNR = 1), the values of MDL indicate that a two-mode model is practically appropriate. This is because, in practice, new modes are not included in the modal sweep process if the MDL or AIC value gets larger. Figure 3 shows that the response given by the identified model fits the true response very well in spite of the presence of high-level noise (SNR = 1). Power spectra of the roof acceleration response for the case with SNR = 1 are compared in Fig. 4, which indicates further that identification results are good in the frequency range of interest. It is remarkable that, although the proposed modal parameter identification approach based on evolutionary programmimg is not computationally more efficient than that using conventional optimization techniques, it removes the stringent requirement that an accurate initial guess of the frequencies for all modes of interest is needed for reliable identification.[16] Because the proposed methodology allows a parallel search of different solutions at the same time, its efficiency and effectiveness can be further improved, provided that machines for parallel computation are available.

Table 1 Modal identification results obtained by simulated evolution under two noisy conditions

	Frequencies, rad/s		Damping ratio, %		EPF[a]	
Mode	True	Estimated[b]	True	Estimated[b]	True	Estimated[b]
1	6.2832	6.2700	5.0000	5.7564	1.2673	1.3339
		6.2781		7.2205		1.4435
2	18.7111	18.8555	5.0000	4.4602	−0.4068	−0.3678
		18.9207		4.0180		−0.3740
3	30.7246	31.0024	5.0000	4.1437	0.2259	0.2158
		31.1967		7.8485		0.1769
4	42.0280	41.8066	5.0000	7.3195	−0.1429	−0.2040
		41.4685		1.8684		−0.2831
5	52.4035	——	5.0000	——	0.0934	——
6	61.6603	——	5.0000	——	−0.0601	——

[a]EPF, effective participation factor.
[b]Upper values correspond to the case of SNR = 5 (number of generations evolved = 2422), and lower values correspond to that of SNR = 1 (number of generations evolved = 2291).

Table 2 Values of different criteria for models having different number of modes for the two cases of different noise levels

No. of modes	Normalized error[a]	AIC[a]	MDL[a]
1	3.7029×10^{-1}	666.88	980.45
	6.2860×10^{-1}	943.92	1380.12
2	2.3015×10^{-1}	551.14	831.82
	5.4481×10^{-1}	913.28	1354.29
3	1.7381×10^{-1}	485.27	755.12
	5.0822×10^{-1}	901.50	1355.62
4	1.3728×10^{-1}	430.87	694.99
	4.6161×10^{-1}	882.87	1347.07
5	1.3557×10^{-1}	433.64	717.34
	4.6143×10^{-1}	888.78	1373.95

[a]Upper values correspond to the case of SNR = 5, and lower values correspond to that of SNR = 1.

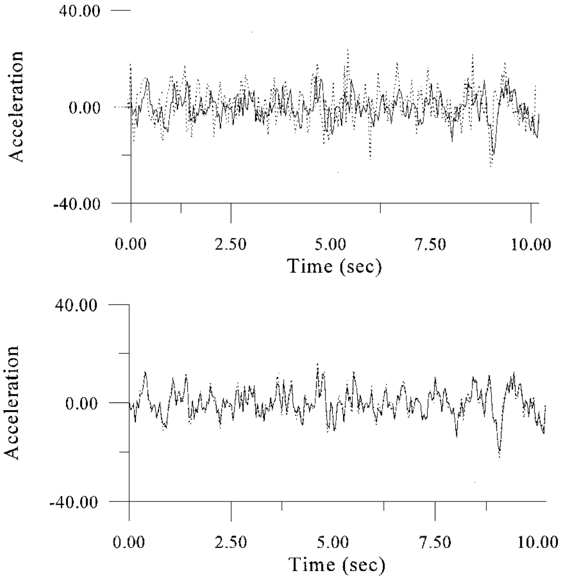


Fig. 3 Comparison between true, measured (SNR = 1) and estimated roof acceleration response ($m \cdot s^{-2}$): ——, true response and ...., measurement (SNR = 1) and estimated response.

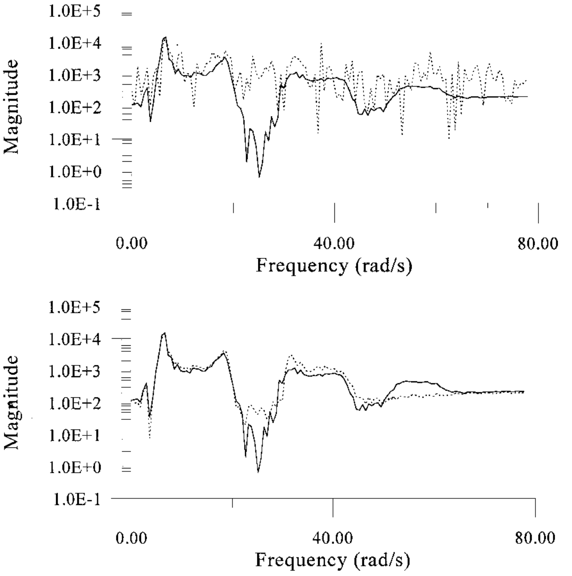


Fig. 4 Comparison between true, measured (SNR = 1) and estimated power spectra of the roof response: ——, true and true power spectrum and ...., measured (SNR = 1) and estimated spectrum.

Conclusions

The method of simulated evolution is implemented and applied to modal parameter identification of linear vibrating structures. An adjustable standard deviation for mutation and a convergence criterion for the evolutionary search based only on the Euclidean distance between the best and the worst organisms (solutions) are proposed. Based on the results obtained in this research, the following conclusions are presented.

1) Results of numerical simulation show that the evolutionary search algorithm is robust to multiple optima. Such robustness is achieved by maintaining a population of candidate solutions at the end of each generation, which actually provides an efficient parallel search of different solutions at the same time.

2) If machines for parallel computation are available, the population size and the number of offspring produced at each generation could be larger for a more effective evolutionary search.

3) Scaling of the standard deviation for random mutation requires some subjective decision. An appropriate choice of standard deviation can significantly accelerate the convergence rate and improve the accuracy of evolutionary search.

4) During the process of modal sweeps, the effect of a dominant mode is extracted before a new mode is included. Using conventional optimization techniques, the search will probably be trapped in an undesired local minimum unless a good initial estimate for parameter values has been made. The evolutionary method does not require such an initial parameter estimate.

5) As shown by the results of numerical simulation, even under tough conditions (SNR = 1), the evolutionary search algorithm demonstrates its capability to locate a good solution. This shows that the proposed modal parameter identification approach using simulated evolution is robust to measurement noise.

6) Further investigations are merited on the applicability of the proposed approach to more realistic problems in which more complicated modes need to be identified.

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